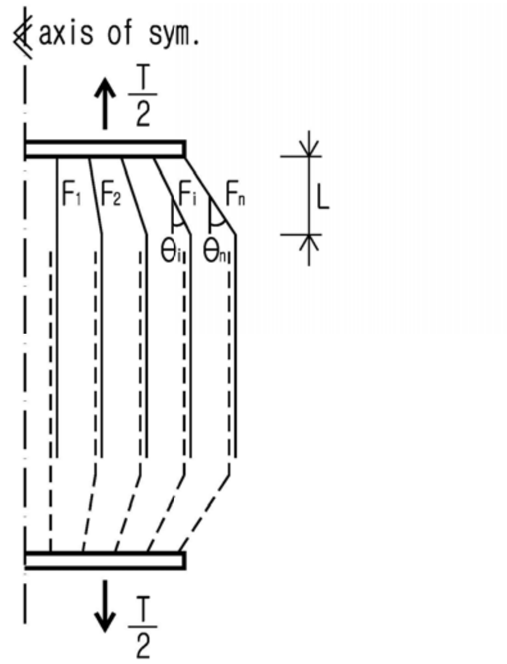
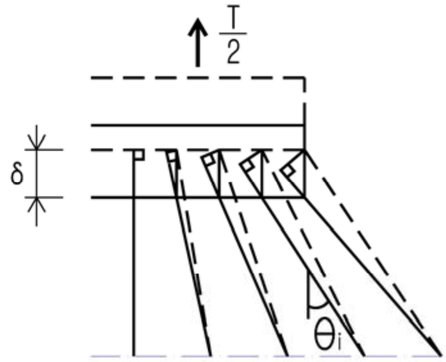
The Friction Secret of interleaved phonebook



1) Normal force from tention



[elongation analysis]

Vertical dispacement is δ
everysheet elongation is $\delta \cdot \cos\theta_i$
everysheet force is F_i

$$F_i = \frac{EA(\delta \cdot \cos\theta_i)}{L/\cos\theta_i} = \frac{EA\delta\cos^2\theta_i}{L}$$

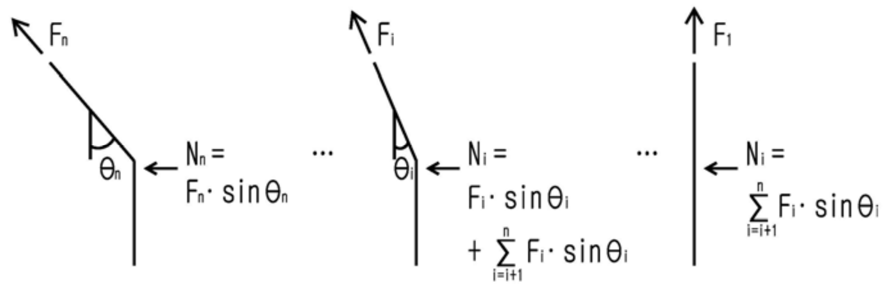
$$T = \sum_{i=1}^{n}(2F_i\cos\theta_i) = \sum_{i=1}^{n} 2\frac{EA\delta\cos^2\theta_i \cdot \cos\theta_i}{L} = \sum_{i=1}^{n}(2\frac{EA\delta\cos^3\theta_i}{L})$$

where E = Young's modulus
A = Sectional area of sheets

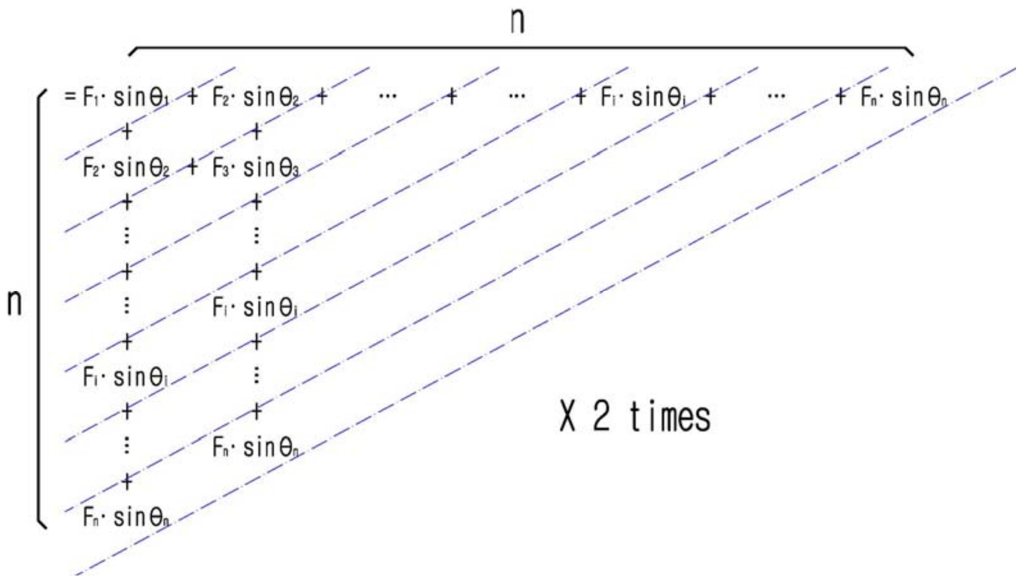
$$\delta = \frac{T}{\sum_{i=1}^{n}\left(\frac{2EA}{L} \cdot \cos^3\theta_i\right)}$$

$$F_i = \frac{EA\delta\cos^2\theta_i}{L} = \frac{EA \cdot T \cdot \cos^2\theta_i}{L \cdot \sum_{i=1}^{n}\left(2\frac{EA}{L} \cdot \cos^3\theta_i\right)} = \frac{T \cdot \cos^2\theta_i}{\sum_{i=1}^{n}\left(2 \cdot \cos^3\theta_i\right)}$$

Normal force(N) on the sheets



$$N = 2 \cdot \sum_{i=1}^{n} N_i = 2\left(\sum_{i=1}^{n} F_i \cdot \sin\theta_i + \sum_{i=2}^{n} F_i \cdot \sin\theta_i + \cdots + \sum_{i=i}^{n} F_i \cdot \sin\theta_i + \cdots + \sum_{i=n}^{n} F_i \cdot \sin\theta_i\right)$$



$$= 2\{n \cdot F_n \cdot \sin\theta_n + (n-1)F_{(n-1)} \cdot \sin\theta_{(n-1)} + \cdots + 1 \cdot F_1 \cdot \sin\theta_1\}$$

$$F_i = \frac{T \cdot \cos^2\theta_i}{\sum_{i=1}^{n} 2 \cdot \cos^3\theta_i}$$

If θ_i is small, $\cos^2\theta_i$ & $\cos^3\theta_i \cong 1.0$, $\sin\theta_i \cong \theta_i$

So, $F_i \cong \frac{T}{2n}$, where $2n$=total number of sheets of each book

If θ_i is spaced equally, $\theta_1 = \theta_1, \theta_2 = 2\theta_1, \cdots, \theta_n = n\theta_1$

$$N \cong 2\left\{n \cdot \left(\frac{T}{2n}\right) \cdot n\theta_1 + (n-1) \cdot \left(\frac{T}{2n}\right) \cdot (n-1)\theta_1 + \cdots + 1 \cdot \left(\frac{T}{2n}\right) \cdot 1\theta_1\right\}$$

$$= 2 \cdot \frac{T}{2n}\left\{n^2 \cdot \theta_1 + (n-1)^2 \cdot \theta_1 + \cdots + 2^2 \cdot \theta_1 + 1^2 \cdot \theta_1\right\}$$

$$= \frac{T}{n} \cdot \frac{n(n+1)(2n+1)}{6} \cdot \theta_1$$

$$= \frac{T}{n} \cdot \frac{(n+1)(2n+1)}{6} \cdot n\theta_1$$

$$= \frac{T}{n} \cdot \frac{(n+1)(2n+1)}{6} \cdot \theta_n$$

2) Friction & Tension Comparison

$$F_f = \mu \cdot N = \mu \cdot \frac{(n+1)(2n+1)}{6n} \cdot \theta_n \cdot T$$

If $\mu \cdot \frac{(n+1)(2n+1)}{6n} \cdot \theta_n \geq 1$, it is self-resistant assembly.